Phase diffusion of Bose-Einstein condensates in a one-dimensional optical lattice

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We investigate the phase diffusion of a Bose-Einstein condensate (BEC) confined in the combined potential of a magnetic trap and a one-dimensional optical lattice. We show that the phase diffusion of the condensate in the whole optical lattice is evident and can be measured from the interference pattern of expanding subcondensates after the optical lattice is switched off. The maximum density of the interference pattern decreases significantly due to the mixing of the phase diffusion appearing in different subcondensates. This suggests a way to detect experimentally the notable phase diffusion of BECs.

I. INTRODUCTION

The remarkable experimental realization of Bose-Einstein condensation has generated intensive theoretical and experimental investigations for weakly interacting Bose gases [1,2]. Although much progress has been made, one of the challenging problems to be solved in both theory and experiment is how to detect the phase diffusion of a Bose-Einstein condensate (BEC). The study of the phase diffusion is of crucial importance because the number of particles realized in all BEC experiments is finite. Within the formalism of the mean-field theory, it is known that the condensate can be well described by a macroscopic wave function (or called an order parameter) with a single phase [1]. The development of the phase at low temperature is governed by the Gross-Pitaevskii (GP) equation. However, for a finite system and hence for trapped Bose gases, although a macroscopic wave function describing a BEC still exists, as is now firmly established experimentally, the phase of the macroscopic wave function will undergo a quantum diffusion process as time develops [3]. Because the phase diffusion reflects directly the coherent nature of a condensate, the study of the phase diffusion of BECs is not only fundamental but also important for applications such as the realization of an atom laser [4].

For a condensate confined in a trap, the development of its phase is determined by the chemical potential of the system. Due to quantum and thermal fluctuations, the phase diffusion is described by the fluctuations of the chemical potential. Even at initial state the condensate has an exact single phase, the phase will become unpredictable due to the fluctuations of the condensate. After the realization of Bose-Einstein condensation, much theoretical attention has been paid to the phase diffusion of BECs [3–13]. Because for a single condensate it is hard to observe the effect of phase diffusion based on the density distribution of a Bose-condensed gas, in the experiment conducted by the JILA group [14] two expanding condensates are used to produce an interference pattern and hence the phase diffusion of the condensates can be detected. It was found that in this experiment there is no detectable phase diffusion on the time scale up to 100 ms.

Recently, the BECs in optical lattices have attracted increasing attention (see Refs [15–19] and references therein). Trapping atoms in optical lattices can provide very effective and powerful means for controlling the properties of a BEC, especially the quantum phase effects on a macroscopic scale such as the coherence of matter waves [17,19]. In the present work, we investigate the effect of the phase diffusion of a BEC confined in the combined potential of a harmonically magnetic trap and a one-dimensional (1D) optical lattice. In the presence of the 1D optical lattice, there are an array of disk-shaped subcondensates. For each subcondensate, i. e. the BEC formed in the corresponding potential well induced by the optical lattice, there exist phase fluctuations which increase very slowly with the development of time. For high enough potential-well intensity, the phase fluctuations for different subcondensates are independent from each other. But when the potential-well intensity is not high, there will be a correlation for the subcondensates in different wells and hence the situation may be quite different. One expects that the phase diffusion of the BEC in the whole optical lattice can display notable new characters because of the mixing effect of the phase diffusion appearing in different subcondensates.

The paper is organized as follows. Section II is devoted to the phase diffusion of a single 2D (as well as quasi-2D) condensate for the temperature far below the critical temperature. In Sec. III, the phase diffusion of an array of disk-shaped subcondensates induced by a 1D optical lattice is investigated based on the interference pattern resulting from a superposition of expanding subcondensates in different potential wells for different holding time of the optical lattice. Sec. IV contains a discussion and summary of our results.

II. PHASE DIFFUSION OF A QUASI-2D CONDENSATE

We consider an array of subcondensates formed in the combined potential consisting of a harmonically magnetic trap and a periodic optical lattice. The combined potential is described by [20–22]:

$$V = \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_\perp^2 \left(y^2 + z^2\right) + sE_R \sin^2\left(\frac{2\pi x}{\lambda}\right), \tag{1}$$

where ω_x and $\omega_\perp$ are respectively the axial and radial frequencies of the magnetic trap. The last term represents the potential due to the presence of the optical lattice, with λ denoting the wavelength of the retroreflected laser beam, and s being a factor characterizing the intensity of the optical potential which increases when increasing the intensity of the laser beam. In addition, $E_R = 2\pi^2\hbar^2/(m\lambda^2)$ is the recoil energy of an atom absorbing one photon. For the optical lattice created by the retroreflected laser beam, $d = \lambda/2$ is the distance between two neighboring wells induced by the optical lattice.

The optical potential can be approximated as $V_{eff} = \sum_k \frac{1}{2} m\widetilde{\omega}_x^2 (x - kd)^2$ with $\widetilde{\omega}_x = 2\sqrt{s}E_R/\hbar$ [20]. As in Ref. [20] we assume that for the parameters appearing in the potential (1) the condition $\omega_x << \omega_\perp << k_B T < \widetilde{\omega}_x$ is satisfied. Thus the magnetic trap is a slowly-varying 3D potential which confines the atoms harmonically to overall cigar-shaped distribution. In the 1D optical lattice the atoms are confined to an array of 2D planes. As a result the whole condensate consists of an array of quasi-2D subcondensates like disks in shelf. Such array of quasi-2D subcondensates has been realized in the experiment [20].

The investigation of the phase diffusion of the abovementioned BEC can be divided into two steps. The first step is to consider the phase diffusion of a single disk-shaped BEC. For this aim we develop a general approach to calculate the probability distribution of the phase for a condensate confined in a quasi-2D harmonic trap. For the temperature far below the critical temperature T_c, the condensate can be described by the order parameter with a phase factor $\phi(t)$:

$$\Phi(\mathbf{r}, t) = \Phi_0(\mathbf{r})\, e^{-i\phi(t)}. \tag{2}$$

The differential of the phase takes the form

$$d\phi = \mu(N_0, T)\, dt/\hbar, \tag{3}$$

where $\mu(N_0, T)$ is the chemical potential of the system. We see that the particle-number fluctuations of the condensate yield fluctuations in the chemical potential, and hence lead to the phase diffusion of the condensate. Assuming that the mean ground state occupation number is $\langle N_0\rangle$, in the case of the particle-number fluctuations $\delta N_0 << N_0$, the average phase of the condensate is then given by

$$\phi(\langle N_0\rangle, t) = \mu(\langle N_0\rangle, T)\, t/\hbar. \tag{4}$$

The phase diffusion of the condensate can be described by considering the phase difference $\Delta\phi(t) = \phi(t) - \phi(\langle N_0\rangle, t)$. From Eqs. (3) and (4), it is straightforward to obtain a differential equation on $\Delta\phi(t)$:

$$\frac{d\Delta\phi(t)}{dt} = \frac{\partial\mu(\langle N_0\rangle, T)}{\partial\langle N_0\rangle}\Delta N_0(t)/\hbar. \tag{5}$$

In the above expression, $\Delta N_0(t)$ represents the fluctuations of the ground state occupation number around $\langle N_0\rangle$ at time t. Different from the situation considered by Jin et al [23] where a small time-dependent perturbation is used artificially to create selected collective excitations in a BEC, the collective excitations considered here are created spontaneously from the condensate due to quantum fluctuations. Thus $\Delta N_0(t)$ changes with the development of time due to the creations and annihilations of various collective modes.

For the temperature far below the critical temperature, the collective excitations spontaneously created from the condensate play a dominant role in the fluctuations of the condensate [13]. We therefore pay attention here to the phase diffusion contributed by the collective excitations of the system. For a pure 2D Bose-condensed gas, the collective mode is labelled by two parameters $\{n, l\}$. Here, the parameter l labels the angular momentum of the excitation. In this case, one has $\Delta N_0(t) = \Sigma_{nl}\Delta N_{nl}(t)$. Assuming that $\Delta\phi_{nl}(t)$ represents the phase diffusion due to the collective mode nl, we have

$$\frac{d\Delta\phi_{nl}(t)}{dt} = \frac{\partial\mu(\langle N_0\rangle, T)}{\partial\langle N_0\rangle}\Delta N_{nl}(t)/\hbar. \tag{6}$$

Thus the overall phase fluctuations of the condensate read $\Delta\phi(t) = \Sigma_{nl}\Delta\phi_{nl}(t)$.

In the above equation, $|\Delta N_{nl}|$ can be regarded as the mean number of atoms corresponding to the collective mode nl. Based on the Bogoliubov theory [24], $|\Delta N_{nl}|$ is given by

$$|\Delta N_{nl}| = \left(\int u_{nl}^2(\mathbf{r})dV + \int v_{nl}^2(\mathbf{r})dV\right) f_{nl} + \int v_{nl}^2(\mathbf{r})dV, \tag{7}$$

where $f_{nl} = 1/\left(\exp(\beta\varepsilon_{nl}) - 1\right)$ is the number of the collective excitations presenting at thermal equilibrium, $\beta = 1/(k_B T)$ (T is temperature), and $\varepsilon_{nl} = \hbar\omega_{nl}$ is the energy spectrum of the collective mode nl. In the above equation, the quantity $\int v_{nl}^2(\mathbf{r})dV$ describes the effect of quantum depletion, which does not vanish even at $T = 0$. In Eq. (7), $u_{nl}(\mathbf{r})$ and $v_{nl}(\mathbf{r})$ are determined by the following coupled equations:

$$\left(-\frac{\hbar^2}{2m}\bigtriangledown^2+V_{\perp}(\mathbf{r})-\mu+2gn(\mathbf{r})\right)u_{nl}(\mathbf{r})$$

$$+gn_0(\mathbf{r})v_{nl}(\mathbf{r})=\varepsilon_{nl}u_{nl}(\mathbf{r}),$$

$$\left(-\frac{\hbar^2}{2m}\bigtriangledown^2+V_{\perp}(\mathbf{r})-\mu+2gn(\mathbf{r})\right)v_{nl}(\mathbf{r})$$

$$+gn_0(\mathbf{r})u_{nl}(\mathbf{r})=-\varepsilon_{nl}v_{nl}(\mathbf{r}),\qquad(8)$$

where $n(\mathbf{r})$ and $n_0(\mathbf{r})$ are the density distributions of the Bose gas and condensate, respectively. $V_{\perp}(\mathbf{r})=\frac{1}{2}m\omega_{\perp}^2(y^2+z^2)$ is the 2D harmonic potential confining the Bose gas in y and z directions (the second term on the right hand side of (1)), while g is the coupling constant describing the interatomic interaction in the condensate.

By a straightforward calculation the excitation frequency of the collective mode nl is given by

$$\omega_{nl}=\omega_{\perp}\left(2n^2+2n\left|l\right|+2n+\left|l\right|\right)^{1/2},\qquad(9)$$

while $|\Delta N_{nl}|$ reads

$$\left|\Delta N_{nl}\right|=\frac{\left(N_0mg\right)^{1/2}\beta_{nl}}{2\sqrt{\pi}\hbar\sqrt{2n^2+2n\left|l\right|+2n+\left|l\right|}},\qquad(10)$$

where

$$\beta_{nl}=\int_0^1\left(1-x^2\right)x\left(H_{nl}\right)^2dx/\int_0^1x\left(H_{nl}\right)^2dx,\qquad(11)$$

$$H_{nl}(x)=x^{|l|}\sum_{j=0}^{n}b_jx^{2j}\qquad(12)$$

with $b_0=1$ and the coefficients b_j satisfying the recurrence relation $b_{j+1}/b_j=\left(4j^2+4j+4j|l|-4n^2-4n-4n|l|\right)/\left(4j^2+4j|l|+8j+4|l|+4\right)$.

As mentioned above, the collective excitations considered here are created spontaneously due to the quantum fluctuations of the condensate. They can disappear as time develops and hence have a finite longevity. The longevity can be estimated based on the time-energy uncertainty relation. For the collective mode nl, its longevity is $\tau_{nl}=1/\omega_{\perp}\left(2n^2+2n\left|l\right|+2n+\left|l\right|\right)^{1/2}$. Thus at time $t>>\tau_{nl}$, there is a series of $i=t/\tau_{nl}$ successive creations and annihilations of the collective mode nl. Assuming that at the time t the number of the collective modes created (annihilated) in the condensate is i_{cre} (i_{ann}). Thus one has $i=i_{cre}+i_{ann}$. In this situation, the desired probability is given by the binomial expression [25]

$$P_i(j)=\left(\frac{1}{2}\right)^i\frac{i!}{\left\{\frac{1}{2}(i+j)\right\}!\left\{\frac{1}{2}(i-j)\right\}!},\qquad(13)$$

where $j=i_{cre}-i_{ann}$. For $t>>\tau_{nl}$, using the Stirling's formula, the asymptotic form of the probability distribution $P_{nl}(\Delta\phi_{nl},t)$ of the phase diffusion due to the collective mode nl takes the form

$$P_{nl}(\Delta\phi_{nl},t)=\frac{1}{\Gamma_{nl}\sqrt{2\pi t/\tau_{nl}}}\exp\left(-\frac{\tau_{nl}\left(\Delta\phi_{nl}\right)^2}{2\Gamma_{nl}^2t}\right),\qquad(14)$$

where the dimensionless parameter Γ_{nl} is given by

$$\Gamma_{nl}=\frac{\partial\mu\left(\langle N_0\rangle,T\right)}{\partial\langle N_0\rangle}\frac{\left|\Delta N_{nl}\right|\tau_{nl}}{\hbar}.\qquad(15)$$

From the Gaussian distribution given by Eq. (14), the phase fluctuations due to the collective mode nl have the form:

$$\left(\delta^2\phi\right)_{nl}=\Gamma_{nl}^2t/\tau_{nl}.\qquad(16)$$

Assuming that the collective modes with different nl are created and annihilated independently, one obtains that the probability distribution of the overall phase difference $\Delta\phi(t)$ is still a Gaussian distribution function. The probability distribution of $\Delta\phi$ is then given by

$$P(\Delta\phi,t)=\sqrt{\frac{B}{\pi}}\exp\left(-B\left(\Delta\phi\right)^2\right).\qquad(17)$$

Based on the general probability theory, the parameter B can be obtained through the relation $\delta^2\phi=1/2B=\Sigma_{nl}\left(\delta^2\phi\right)_{nl}$. From the formulas (10), (15), (16) and the chemical potential $\mu(N_0,T)=\left(N_0mg/\pi\right)^{1/2}\omega_{\perp}$ for 2D harmonic trap, we obtain the overall phase fluctuations $\delta^2\phi$ as

$$\delta^2\phi=\frac{m^2g^2\omega_{\perp}t}{4\pi^2\hbar^4}\Upsilon,\qquad(18)$$

where the coefficient Υ takes the form

$$\Upsilon=\sum_{nl}\frac{\beta_{nl}^2}{\left(2n^2+2n\left|l\right|+2n+\left|l\right|\right)^{3/2}}\left[f_{nl}+\frac{1}{2}\right]^2,\qquad(19)$$

where β_{nl} has been given in Eq. (11). From Eq. (18) we see that when the longevity of the collective excitations is taken into account, the phase fluctuations are proportional to the time t, rather than t^2 which is obtained when the collective excitations are regarded as quite stable.

As mentioned above, if $\omega_x<<\omega_{\perp}<<k_BT<\widetilde{\omega}_x$, the condensate confined in each well has a quasi-2D nature. Note that for the array of the subcondensates induced by the optical lattice with enough depth, the overlap between the subcondensates occupying different wells can be omitted. This means that the behavior of the condensate in each well has a local property. For each well

the coupling constant g is given by $g \approx 2\sqrt{2\pi}\hbar^2 a_s/(m\widetilde{l_x})$ [26], which is fixed by a s-wave scattering length a_s and the oscillator length $\widetilde{l_x} = (\hbar/m\widetilde{\omega}_x)^{1/2}$ in the x-direction. Thus from Eq. (18) the subcondensate in each well displays the phase fluctuations

$$\delta^2\phi = \frac{2\Upsilon\omega_\perp t}{\pi}\left(\frac{a_s}{\widetilde{l_x}}\right)^2. \tag{20}$$

From the above result, one has $\delta^2\phi \sim \omega_\perp\widetilde{\omega}_x$. We see that the confinement induced by the optical lattices has the effect of increasing the phase fluctuations. This is a natural character by considering the fact that the confinement has the effect of increasing the particle-number fluctuations [27].

III. EFFECT OF PHASE DIFFUSION ON THE INTERFERENCE PATTERN OF EXPANDING ARRAY OF SUBCONDENSATES

We now go to the second step, i. e. to discuss the phase diffusion of an array of subcondensates confined in the combined potential of the harmonically trapping potential and 1D optical lattice. To illustrate clearly the role of the phase diffusion in the interference pattern of the expanding subcondensates, we consider here an experiment scheme which can be realized in future experiment. Firstly, a cigar-shaped condensate is formed in a magnetic trap with the harmonic angular frequencies $\omega_x = 2\pi\times 9$ Hz and $\omega_\perp = 2\pi\times 92$ Hz at a temperature far below the critical temperature. The atoms are transferred into the lattice potential by increasing the power of the laser beam so that the harmonic angular frequency of the well is approximated to be $2\pi\times 14$ kHz in $x-$direction. For $\widetilde{\omega}_x = 2\pi\times 14$ kHz, based on the well-known Bose-Hubbard model, the tunnelling time between neighboring subcondensates is of the order of 1 ms. In this situation, the array of subcondensates should be regarded as fully coherent and there is no independent random phase for the subcondensates in different wells. Although there is still a phase diffusion for the whole condensate, one can not observe this through the interference pattern. To observe the effect of the phase diffusion, one can rapidly increase the lattice potential depth to a value of $\widetilde{\omega}_x = 2\pi\times 52$ kHz within a time (for example 50 μs) much smaller than the tunnelling time. In this situation, the phase of every subcondensate is the same after the lattice potential ramps to its final strength. For the lattice potential with $\widetilde{\omega}_x = 2\pi\times 52$ kHz, the tunnelling time between neighboring wells is of the order of 1.5 s. Thus, for the holding time t_h of the optical lattice being much smaller than 1.5 s, the phase diffusion of the subcondensates in different wells can be regarded as independent from each other. In this situation, there is a random phase for each subcondensate with the development of the holding time and this will lead to an obvious effect on the interference pattern after the combined potential is switched off.

For the experimental parameters considered here, we have $\delta^2\phi = 1.60\times t_h$. Based on the result given by Eq. (20), the probability distribution of the phase for the subcondensate in each well can be obtained through Eq. (17). For the interference pattern to wash out completely, the time scale of the holding time is obtained by using $\delta^2\phi = \pi^2$. In the experimental parameters used here, for the subcondensate in each well $\delta^2\phi$ is still much smaller than π^2 even when the holding time t_h of the optical lattice is 1s. Due to the fact that there are many subcondensates induced by the optical lattice, however, we will show that there is a significant effect of the phase diffusion to the interference pattern after the optical lattice is switched off.

To observe a notable effect of the phase diffusion, we consider the case when only the optical lattice is switched off after the holding time $t_h = 0.2$ s. Assume t_0 is the time after the optical lattice is switched off. For the subcondensate confined in the kth well of the optical lattice, applying the propagator method used in Ref. [28] it is easy to get the normalized wave function $\varphi_k(x,t_0)$ after only the optical lattice is switched off:

$$\varphi_k(x,t_0) = A_n\sqrt{\frac{1}{\sin\omega_x t_0\left(\mathrm{ctg}\omega_x t_0 + i\gamma\right)}}\exp[iR_k(\delta\phi,t_h)]$$
$$\times\exp\left[-\frac{(kd\cos\omega_x t_0 - x)^2}{2\sigma^2\sin^2\omega_x t_0\left(\mathrm{ctg}^2\omega_x t_0+\gamma^2\right)}\right]$$
$$\times\exp\left[-\frac{i\,(kd\cos\omega_x t_0 - x)^2\,\mathrm{ctg}\omega_x t_0}{2\gamma\sigma^2\sin^2\omega_x t_0\left(\mathrm{ctg}^2\omega_x t_0+\gamma^2\right)}\right]$$
$$\times\exp\left[\frac{i\left(x^2\cos\omega_x t_0 + k^2d^2\cos\omega_x t_0 - 2xkd\right)}{2\gamma\sigma^2\sin\omega_x t_0}\right], \tag{21}$$

where $A_n = 1/\pi^{1/4}\sigma^{1/2}$ is a normalization constant, the dimensionless parameter $\gamma = \hbar/m\omega_x\sigma^2$ with σ the width of the subcondensate in each well [28]. The factor $R_k(\delta\phi,t_h)$ represents a random phase due to the phase diffusion of the subcondensate in the kth well. In numerical calculations, $R_k(\delta\phi,t_h)$ is generated according to the probability distribution function $P(\Delta\phi,t_h)$ (see (17)). Then we obtain the density distribution in $x-$direction as follows:

$$n(x,t_0) = \Xi\left|\sum_{k=-k_M}^{k_M}\left(1-\frac{k^2}{k_M^2}\right)\varphi_k(x,t_0)\right|^2, \tag{22}$$

where $\Xi = 15N_{all}k_M^3/\left(16k_M^4-1\right)$. N_{all} is the total number of atoms in the array of condensates and $2k_M+1$ is

the total number of the subcondensates induced by the optical lattice. In the present work, by using the experimental result in [20], k_M is chosen to be 100.

It is obvious that the density distribution $n(x, t_0)$ at $x = 0$ reaches the maximum value at time $t_l = (2l-1)\pi/2\omega_x$, where l is a positive integer. Displayed in Fig. 1 is $n(x, t_0)$ (with $N_{all}\Xi A_n^2$ as a unit) at t_l with the holding time $t_h = 0.2$ s. We see that the phase diffusion of the array of the subcondensates makes the central density of the interference pattern decrease significantly. Displayed in Fig. 2 is the density distribution $n(x = 0, t_0 = t_l)$ versus the holding time t_h. A exponential damping of the central density is clearly shown in the figure.

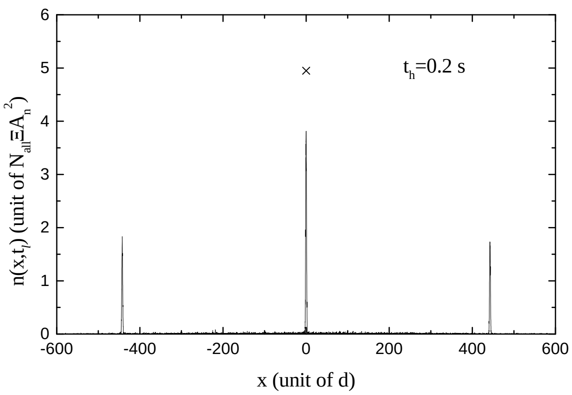

FIG. 1. Displayed is the density distribution of the interference pattern at $t_0 = t_l$ for the case of the holding time t_h of the optical lattices being 0.2 s. The density distribution $n(x, t_l)$ is in unit of $N_{all}\Xi A_n^2$, while the coordinate x is in unit of d. In this figure, the position of the cross $\times$ indicates the density at $x = 0$ in the case of $t_h = 0$ s. The figure shows clearly that the phase diffusion has the effect of decreasing significantly the maximum density of the central peak.

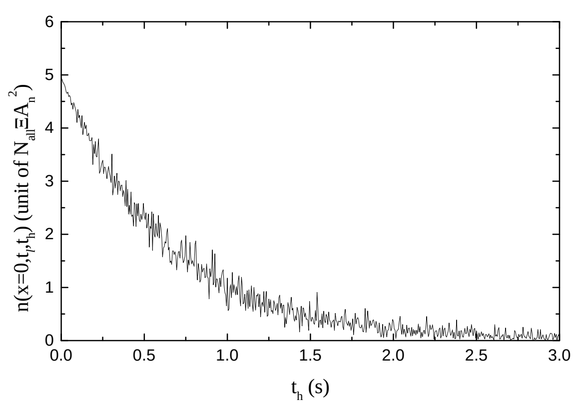

FIG. 2. Shown is $n(x = 0, t_l, t_h)$ versus the holding time t_h. The exponential damping of the central peak is clearly shown in the figure.

It is straightforward to calculate the interference pattern of the expanding array of BECs when the magnetic trap and optical lattice are both switched off. Shown in Fig. 3 is the interference pattern for the holding time $t_h = 0.2$ s after the combined potential is switched off for 30 ms. In comparison with the case of $t_h = 0$ s (dashed line in Fig. 4), the strong noise in Fig. 3 shows that the phase diffusion has an important effect on the interference pattern of the array of subcondensates. The solid line in Fig. 4 displays the average density distribution of twenty ensembles for the holding time $t_h = 0.2$ s. We see that the maximum density of the central and side peaks decreases significantly due to the presence of the phase diffusion. Generally speaking, there are four parameters $\omega_\perp$, $\widetilde{\omega}_x$, k_M, t_h which are related closely to the effect of the phase diffusion. The present work shows that increasing these parameters will contribute to the observation of the phase diffusion for BEC in optical lattices.

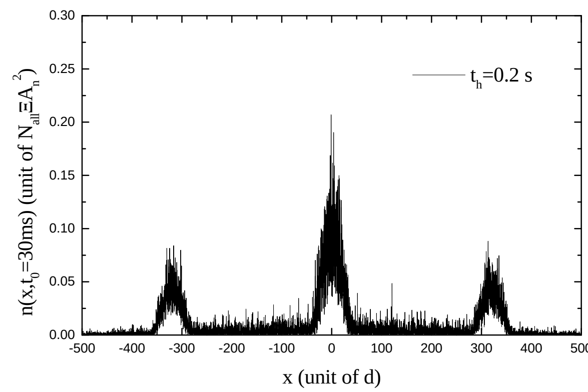

FIG. 3. After the holding time 0.2 s of the optical lattices, when both the magnetic trap and optical lattices are switched off, the density distribution of the expanding condensates is shown for $t_0 = 30$ ms. In the figure, $n(x, t_l)$ is in unit of $N_{all}\Xi A_n^2$, while x is in unit of d. Compared with the density distribution of the zero holding time (dashed line in Fig. 4), the effect of the phase diffusion on the interference pattern is very obvious.

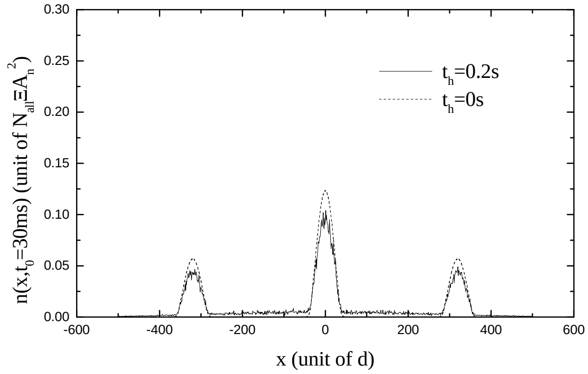

FIG. 4. The solid line displays the average density distribution of twenty ensembles when both the magnetic trap and optical lattices are switched off for $t_0 = 30$ ms in the case of the holding time $t_h = 0.2$ s. The dashed line shows the interference pattern in the case of the holding time $t_h = 0$.

IV. DISCUSSION AND SUMMARY

Based on the analysis of the collective excitations, we have investigated the phase diffusion at low temperature in an array of subcondensates formed in a combined potential consisting of harmonically magnetic trap and 1D optical lattice. We have shown that the effect of phase diffusion on the interference pattern is evident and thus can be measured from the expanding subcondensates after the optical lattice is switched off. We point out that the maximum density of the interference pattern decreases significantly due to the mixing of the phase diffusion appearing in different subcondensates formed by the optical lattice. This suggests a way to detect experimentally the notable phase diffusion of BEC. For the case of BEC in 3D optical lattice, however, it seems that the theory developed here can not be used due to the fact that there are only an average atom number of up to 2.5 atoms per lattice site [19].

Note that a recent experiment has been conducted in Ref. [21] where the interference pattern is measured when only the optical lattice is switched off. Clearly this type of experiment can be used to test the theoretical predictions provided in this paper. We stress that for a single subcondensate the phase fluctuations increase very slowly with the development of the time. However, the phase diffusion plays an important role in the interference pattern of the expanding subcondensates, as shown in the present work.

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